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Finite Element Analysis of a Beam with Piezoelectrics using Third Order Theory—Part II Dynamic Analysis-Active Vibration Control

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ABSTRACT : In recent year, considerable amount of work has been done on vibration control of Smart structures using piezoelectrics. Active vibration control of a fixed beam using surface bonded piezoelectric sensors and actuators is examined in this work. The finite element model developed is based on Reddy's third order laminate theory. The simulation results show that an increase in the number of sensor/actuator pairs improves the vibration control of the beam. However, the location of the sensors/actuators is even more important in controlling active vibrations. The sensors/actuators pairs when placed near the regions of highest strains give the best vibration suppression and show little effect in the lowest strain regions.

Keywords : Smart Structure, Piezoelectric sensors and actuators, Fixed ends beam, Shape control, Vibration control.

I. INTRODUCTION

The ability to respond automatically to changes in their environment, smart structures offer a simplified approach to the control of various material and system characteristics such as noise, shape and vibration, etc. Monitoring and control of vibrations is vital in achieving the desired objectives of many engineering systems. A few applications are vibration suppression of aircraft structures, noise control of helicopter rotors, health monitoring of bridges, shape control of large space trusses, aero-elastic control of aircraft lifting components and seismic control of buildings. Advances in smart materials technology have produced much smaller actuators and sensors with high integrity in structures and an increase in the application of smart materials for passive and active structural damping.

Several investigators have developed analytical and numerical, linear and non-linear models for the response of integrated piezoelectric structures. These models provide platform for exploring the shape and active vibration control in smart structures. The experimental work of Bailey and Hubbard, 1985 [1] is usually cited as the first application of piezoelectric materials as actuators. They successfully used piezoelectric sensors and actuators in the vibration control of isotropic cantilever beams. Crawley and de Luis, 1987 [2] formulated static and dynamic analytical models for extension and bending in beams with attached and embedded piezoelectric actuators. Heyliger and Reddy, 1988 [3] developed a finite element model for bending and vibration problems using third order shear deformation theory. They successfully used piezoelectric sensors and actuators in the vibration control of isotropic cantilever beams based on the classical laminated plate theory. Ha, Keilers and Chang,1992 [4] developed a three-dimensional brick element to model the dynamic and static response of laminated composites containing distributed piezoelectrics, and then studied the active response control for the integrated structures by coupling simple control algorithms in a closed loop. Chandrashekhara and Varadarajan, 1997 [5] gave a finite element model based on higher order shear deformation theory for laminated composite beams with integrated piezoelectric actuators. Valoor et al, 2000 [6] used neural network-based control system for vibration control of laminated plates with piezoelectrics. Lee and Reddy, 2004 [7] used the third-order shear deformation theory to control static and dynamic deflections of laminated composite plates. Prasad et al, 2005 [8] developed a criterion for the evaluation and selection of piezoelectric materials and actuator configurations.

The accuracy and efficiency of active vibration control or suppression models depend on the perfection of understanding the mechanical interaction between the piezoelectrics and the underlying structure. The Euler-Bernoulli classical theory used to model the beam/plate deformation neglects the transverse shear deformation effects. The shear deformation theory has a disadvantage as it needs a shear correction factor, which is very difficult to determine especially for arbitrarily laminated composite structures with piezoelectric layers. To overcome the above-mentioned drawbacks, Reddy,1984 [9] developed a third order laminate theory, which takes into account the quadratic variation of transverse shear strains, eliminates the transverse shear stresses on the top and bottom of a laminated composite structure Thus, no shear correction factor is needed in the third order theory.

II. PIEZOELECTRIC EQUATIONS

Assuming that a beam consists of a number of layers (including the piezoelectric layers) and each layer possesses a plane of material symmetrically parallel to the *x-y* plane and a linear piezoelectric coupling between the elastic field and the electric field the constitutive equations for the layer can be written as,

$$\begin{Bmatrix} D_1 \\ D_3 \end{Bmatrix}_k = \begin{bmatrix} 0 & e_{15} \\ e_{31} & 0 \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_5 \end{Bmatrix}_k + \begin{bmatrix} \bar{\varepsilon}_{11} & 0 \\ 0 & \bar{\varepsilon}_{33} \end{bmatrix}_k \begin{Bmatrix} E_1 \\ E_3 \end{Bmatrix}_k \quad \text{... (1)}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_5 \end{Bmatrix}_k = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_5 \end{Bmatrix}_k - \begin{bmatrix} 0 & e_{31} \\ e_{15} & 0 \end{bmatrix}_k \begin{Bmatrix} E_1 \\ E_3 \end{Bmatrix}_k \quad \text{... (2)}$$

The thermal effects are not considered in the analysis. The piezoelectric constant matrix [e] can be expressed as

$$[e] = [d][Q] \quad \text{... (3)}$$

where,

$$[d] = \begin{bmatrix} 0 & d_{15} \\ d_{31} & 0 \end{bmatrix} \quad \text{... (4)}$$

III. NOMENCLATURE

a, b	Constants
$[C]$	Global damping matrix
$[C^*]$	Control algorithm damping matrix
$[C^e]$	Elemental damping matrix
$[d]$	Piezoelectric strain constant matrix
D	Electric Displacement field
$[e]$	Piezoelectric constant matrix
e	Piezo electric constant
E	Electric field; Young's Modulus of elasticity
$\{F_v\}$	Global electrical force vector
$\{F\}$	Global external mechanical force
G_c	Gain of the current amplifier vector
$[G]$	Control gain matrix
G_i	Gain to provide feedback control
$[K]$	Global Stiffness matrix
$[K^c]$	Elemental Stiffness matrix
$[M]$	Global mass matrix
$[M^e]$	Elemental mass matrix
N_i	Shape function the i^{th} element
Q	General Stiffness of the material
S_i	Strain energy of the j^{th} element
t	Total thickness of the beam
u, v, w	Displacements of a point along x, y and z directions respectively
V	Applied voltage to Piezo actuator
V_S	Open circuit sensor voltage
u_0, w_0	Displacement of a point on the mid-plane along the x and z direction respectively
$\{\bar{u}\}$	Nodal displacement vector
$\{\ddot{\bar{u}}\}$	Nodal acceleration vector
$\{x\}$	Generalized displacements
x, y, z	Cartesian coordinates
ϕ_x	Bending rotation of x-axis
ε_i	Strain of i^{th} element in strain tensor
$\bar{\varepsilon}$	Absolute permittivity of the dielectric
$\{\sigma\}$	stress vector
$\{\varepsilon\}$	strain vector
ϕ	Rotation of the transverse normal about y-axis
φ	Cubic Hermit interpolation polynomial
Δ_1, Δ_3	Nodal values of ω_0
Δ_2, Δ_4	Nodal values of $\frac{\partial \omega_0}{\partial x}$
ψ	Linear Lagrangian interpolation polynomial
ξ	Model damping ratio
ψ_j^e	Quadratic Lagrange's interpolation functions
	i^{th} natural frequencies
$[\Phi]$	Modal matrix
φ	Cubic Hermit interpolation polynomial
ψ	Linear Lagrangian interpolation polynomial
$[\Omega]$	Diagonal matrix that stores square of the natural frequencies

IV. DISPLACEMENT FIELD OF THE THIRD ORDER THEORY

The displacement field based on the third order beam theory of Reddy [9] is given by

$$u(x,z,t) = u_0(x,t) + z\phi_x(x,t) - \alpha z^3\left(\phi_x + \frac{\partial w_0}{\partial x}\right) \quad \text{... (5)}$$

$$w(x, z, t) = w_0(x, t) \quad \text{... (6)}$$

where $\alpha = \frac{4}{3t^2}$ and t is the total thickness of the beam.

The displacement functions are approximated over each finite element by

$$u_0(x,t) = \sum_{i=1}^{2} u_i(t)\psi_i(x) \quad \text{... (7)}$$

$$\phi_x(x,t) = \sum_{i=1}^{2} \phi_i(t)\psi_i(x) \quad \text{... (8)}$$

$$w_0(x,t) = \sum_{i=1}^{4} \Delta_i(t)\varphi_i(x) \quad \text{... (9)}$$

Using finite element formulation equations (5) and (6) can be expressed as,

$$\{u\} = [N]\{\bar{u}\}$$

$$\{\bar{u}\} = \{u_1 \quad \phi_1 \quad \Delta_1 \quad \Delta_2 \quad u_2 \quad \phi_2 \quad \Delta_3 \quad \Delta_4\}^T \quad \text{... (10)}$$

where,

$$[N]=\begin{bmatrix} \psi_1 & 0 \\ (z-\alpha z^3)\psi_1 & 0 \\ -\alpha z^3\dfrac{\partial\varphi_1}{\partial x} & \varphi_1 \\ -\alpha z^3\dfrac{\partial\varphi_2}{\partial x} & \varphi_2 \\ \psi_2 & 0 \\ (z-\alpha z^3)\psi_2 & 0 \\ -\alpha z^3\dfrac{\partial\varphi_3}{\partial x} & \varphi_3 \\ -\alpha z^3\dfrac{\partial\varphi_4}{\partial x} & \varphi_4 \end{bmatrix} \quad \text{... (11)}$$

The strain-displacement relations are given by

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_1\\ \varepsilon_5\end{Bmatrix}=[B]\{\bar{u}\} \quad \text{... (12)}$$

where,

$$\{B\}=\begin{bmatrix} \dfrac{\partial\psi_1}{\partial x} & 0 \\ (z-\alpha z^3)\dfrac{\partial\psi_1}{\partial x} & (1-3\alpha z^2)\psi_1 \\ -\alpha z^3\dfrac{\partial^2\varphi_1}{\partial x^2} & (1-3\alpha z^2)\dfrac{\partial\varphi_1}{\partial x} \\ -\alpha z^3\dfrac{\partial^2\varphi_2}{\partial x^2} & (1-3\alpha z^2)\dfrac{\partial\varphi_2}{\partial x} \\ \dfrac{\partial\psi_2}{\partial x} & 0 \\ (z-\alpha z^3)\dfrac{\partial\psi_2}{\partial x} & (1-3\alpha z^2)\psi_2 \\ -\alpha z^3\dfrac{\partial^2\varphi_3}{\partial x^2} & (1-3\alpha z^2)\dfrac{\partial\varphi_3}{\partial x} \\ -\alpha z^3\dfrac{\partial^2\varphi_4}{\partial x^2} & (1-3\alpha z^2)\dfrac{\partial\varphi_4}{\partial x} \end{bmatrix}$$

V. EQUATIONS OF MOTION

The dynamic equations of the piezoelectric structure are derived using Hamilton's principle. These equations also provide coupling between electrical and mechanical terms. The electric force due to the applied charge of the actuator is not considered in the analysis. The equation of motion including the damping effects (Rayleigh damping is assumed) can be written as,

$$[M^e]\{\ddot{\bar{u}}\}^e+[C^e]\{\dot{\bar{u}}\}^e+[K^e]\{\bar{u}\}^e=\{F\}^e+[K^e_{uv}]V^e \quad \text{.. (13)}$$

where,

$$[M]=\int_{V_e}\rho[N]^T[N]dV \quad \text{... (14)}$$

$$[K^e]=\int_{V_e}\{B][Q][B]dV \quad \text{... (15)}$$

$$[K^e_{uv}]=\int_{V_e}[B]^T[e]^T[B_v]dV \quad \text{... (16)}$$

$$[C^e]=a[M^e]+b[K^e]$$

Assembling all the elemental equations gives the global dynamic equation,

$$[M]\{\ddot{\bar{u}}\}+[C]\{\dot{\bar{u}}\}+[K]\{\bar{u}\}=\{F\}+\{F_V\} \quad \text{... (17)}$$

where,

$$\{F\} = [K_{uv}]\{V\} \quad \text{... (18)}$$

VI. SENSOR EQUATIONS

Since no external electric field is applied to the sensor layer and as charge is collected only in the thickness direction, only the electric displacement D_3 is of interest and can be written as

$$D_3 = e_{31}\varepsilon_1 \quad \text{... (19)}$$

Assuming that the sensor patch covers several elements, the total charge the total charge developed on the sensor surface is

$$q(t)=\sum_{j=1}^{N_s}\frac{1}{2}\left[\int_{S_j}\left([B_1]_{(z=z_k)}+[B_1]_{(z=z_{k+1})}\right)e_{31}dS\{\bar{u}_j\}\right] \quad \text{.. (20)}$$

where $[B_1]$ is the first row of $[B]$

The distributed sensor generates a voltage when the structure is oscillating; and this signal is fed back into the distributed actuator using a control algorithm, as shown in Fig. 1. The actuating voltage under a constant gain control algorithm can be expressed as,

$$V^e=G_iV_s=G_iG_c\frac{dq}{dt} \quad \text{... (21)}$$

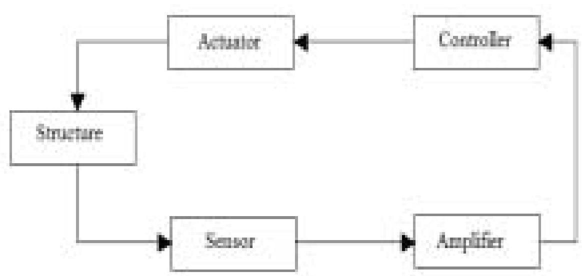

Fig. 1. Block Diagram of Feedback Control System.

The system actuating voltages can be written as

$$\{V\}=[G][K_v]\{\dot{\bar{u}}\} \quad \text{... (22)}$$

where $[G]$ is the control gain matrix and G = G_iG_c.

In the feedback control, the electrical force vector $\{F_v\}$ can be regarded as a feedback force. Substituting equation (22) into equation (18) gives

$$[F_v] = [K_{uv}][G][K_v]\{\dot{\bar{u}}\} \quad \text{... (23)}$$

We define

$$[C^*] = -[K_{uv}][G][K_v] \quad \text{... (24)}$$

Thus, the system equation of motion, equation (28) becomes

$$[M]\{\ddot{\bar{u}}\} + ([C^*] + [C])\{\dot{\bar{u}}\} + [K]\{\bar{u}\} = \{F\} \quad \text{... (25)}$$

Equation (25) shows that, the voltage control algorithm has a damping effect on the vibration suppression of a distributed system.

VI. SYSTEM RESPONSE USING MODAL ANALYSIS

To obtain the dynamic response of the system under a given external loading condition, a modal analysis technique is used. The nodal displacement is given by

$$\{\bar{u}\}[\Phi]\{x\} \quad \text{... (26)}$$

Equation (25), is modified as

$$\{\ddot{x}\} + ([\Phi]^T[C^*][\Phi] + [\Phi]^T[C][\Phi])\{\dot{x}\} + [\Omega]\{x\} = [\Phi]^T\{F\} \quad \text{... (27)}$$

Since, for a particular natural frequency and hence mode shape

$$[\Phi]^T[C][\Phi] = 2\xi\omega \text{ and } [\Omega] = \omega^2$$

Equation (25) becomes,

$$\{\ddot{x}\} + (2\omega\xi + [\Phi]^T[C^*][\Phi])\{\dot{x}\} + \omega^2\{x\} = [\Phi]^T\{F\} \quad \text{... (28)}$$

The initial conditions on $\{x\}$ can be obtained as follows:

$$\{x_0\} = [\Phi]^T[M]\{\bar{u}_0\} \quad \text{... (29)}$$

$$\{\dot{x}_0) = [\Phi]^T[M]\{\dot{\bar{u}}\} \quad \text{... (30)}$$

VII. ACTIVE VIBRATION CONTROL

A beam having both ends fixed with both the upper and lower surfaces bonded by piezoelectric ceramics is shown in Fig. 2. The beam is made of T300/976 Graphite/Epoxy composites and the Piezoceramic is PZT G1195N. The adhesive layers are considered to be of Isotac. The material properties are given in Table1. The total thickness of the beam is 10 mm and the thickness of each Piezoceramic and adhesive layers are 0.2 mm and 0.1 mm respectively. The lower Piezoceramics serve as sensors and the upper ones as actuators. The relative sensors and actuators form sensor/actuator (S/A) pairs through closed control loops.

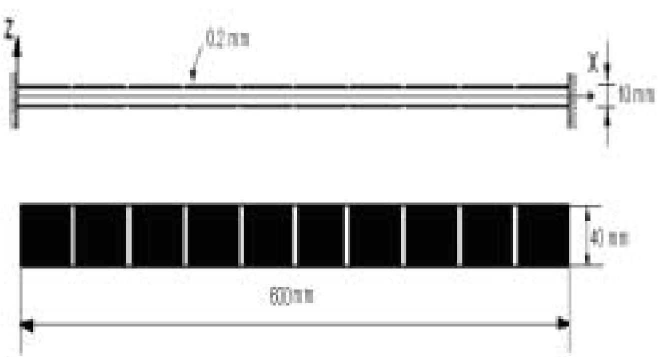

Fig. 2. Fixed ends beam with ten pairs of surface bonded piezoelectric sensors and actuators.

Table 1. Material properties PZT G1195N Piezoceramic and T300/976 Graphite/Epoxy composites and Adhesive layer.

	PZT	T30/976	Isotac
Young's moduli (GPa)			1.1
E_{11}	63.0	150.0	
$E_{22} = E_{33}$	63.0	9.0	
Shear moduli (GPa)			
$G_{12} = G_{13}$	24.2	7.10	
G_{23}	24.2	2.50	
Density, ρ (kg/m3)	7600	1600	890
Piezoelectric constants (m/V)			
$d_{11} = d_{22}$	254×10^{-12}		
Electrical permittivity (F/m)			
$\varepsilon_{11} = \varepsilon_{22}$	15.3×10^{-9}		
ε_{33}	15.0×10^{-9}		
First mode damping coefficient, ξ	---	0.009	

The beam as shown in Fig. 2, is considered to simulate the active vibration control through a simple S/A active control algorithm. The beam is assumed to vibrate freely due to an initial disturbance (first mode) at the middle. The Piezoceramics on the lower surface are used as sensors and those on the upper surface as actuators. For the analysis, the whole beam is evenly divided into 40 elements with each S/A pair covering four elements. The effect of negative velocity feedback control gains on the transient response of the cantilever beam subjected to the first mode vibrations is shown in Figures 3 to 6. Ten S/A pairs covering the full span of the beam are used in active vibration suppression. It can be seen from the figures that, vibrations decay more quickly when higher control gains are applied. However, the gains should be limited for the sake of the breakdown voltage of the piezoelectric materials.

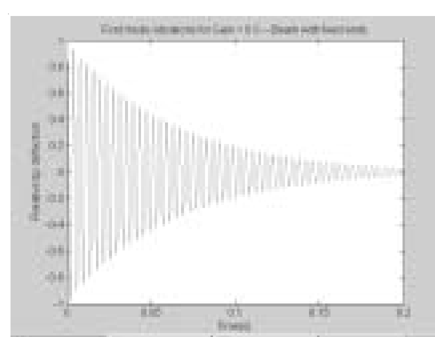

Fig. 3. The effect of negative velocity gain on fixed ends beam subjected to first mode vibrations. (Ten pairs of *S/As* evenly distributed). Gain = 0*V/A*.

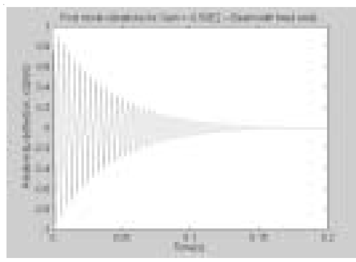

Fig. 4. The effect of negative velocity gain on fixed ends beam subjected to first mode vibrations. (Ten pairs of *S/As* evenly distributed). Gain = -0.5×10^2 *V/A*.

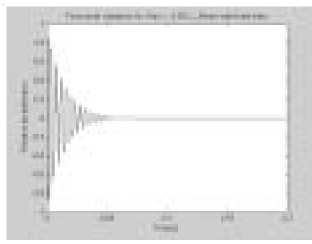

Fig. 5. The centerline deflection of the beam with two pairs of actuators located at the middle span.

Similar to the shape control simulation, four different sets of S/A pairs are considered to evaluate the effect of the number of S/A on the active vibration suppression. The decay curves of the middle point displacement with different pairs of S/A are shown in Figure 6. The control gain of   for all the four sets of S/A is used. Form the figure it is clear that the vibrations decay out more quickly with an increase in the number of S/A pairs but the location of S/A pairs is more important. The position of S/A pairs is very important in vibration control. Similar to the case of shape control, the S/A pairs are not very effective in vibration suppression when located at the fixed ends. From the Fig. 6, it can be seen that the S/A pairs have the best effect on vibration suppression when located at the mid span of the beam.

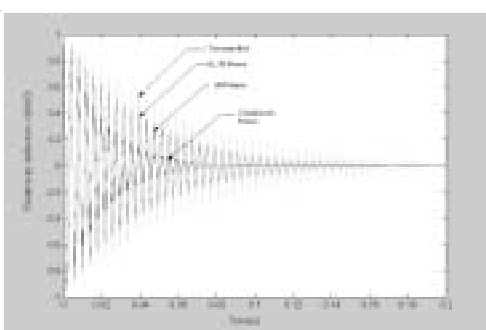

Fig. 6. Decay Curves of the mid point deflection of the fixed ends beam with different pairs of *S/As*. Gain = -2.5×10^2 *V/A*.

VIII. CONCLUSION

An efficient and accurate finite element model and computer codes (in Matlab), based on the third order laminate theory, are developed for the active vibration control of a beam having both ends fixed with distributed piezoelectric ceramics. From the simulation results obtained, it is observed that the number and location of the sensor/actuator is very important in vibration suppression. When the sensor/actuator pairs are placed in high strain regions, they are very effective in controlling the vibrations whereas when they are placed in low strain regions they have little effect on vibration suppression. An increase in the number of sensor/actuator pairs shows better results for controlling vibrations, but their location is found to be more critical. Thus, it can be concluded that the number and location of sensor/actuator pairs must be considered carefully in designing smart structures with distributed piezoelectric sensor/actuator pairs.

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